Looking at Euler flows through a contact mirror: Universality and Turing completeness

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The dynamics of an inviscid and incompressible fluid flow on a Riemannian manifold is governed by the Euler equations. Recently, Tao [6, 7, 8] launched a programme to address the global existence problem for the Euler and Navier-Stokes equations based on the concept of universality. Inspired by this proposal, we show that the stationary Euler equations exhibit several universality features, in the sense that, any non-autonomous flow on a compact manifold can be extended to a smooth stationary solution of the Euler equations on some Riemannian manifold of possibly higher dimension [1]. A key point in the proof is looking at the h-principle in contact geometry through a contact mirror, unveiled by Etnyre and Ghrist in [4] more than two decades ago.

We end up this talk addressing a question raised by Moore in [5] : *"Is hydrodynamics capable of performing computations?"*. The universality result above yields the Turing completeness of the steady Euler flows on a 17-dimensional sphere. Can this result be improved? In [2] we construct a Turing complete Euler flow in dimension 3. Time permitting, we discuss this and other generalizations contained in [3].

This talk is based on several joint works with Cardona, Peralta-Salas and Presas.

References

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