

Testing Arrays for Fault Localization

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A *separating hash family* $\text{SHF}_\lambda(N; k, v, \{w_1, \dots, w_s\})$ is an $N \times k$ array on v symbols, with the property that no matter how we choose disjoint sets C_1, \dots, C_s of columns with $|C_i| = w_i$, there are at least λ rows in which, for every $1 \leq i < j \leq s$, no entry in a column of C_i equals that in a column of C_j . (That is, there are λ rows in which sets $\{C_1, \dots, C_s\}$ are *separated*.) Separating hash families have numerous applications in combinatorial cryptography and in the construction of various combinatorial arrays; typically, one only considers whether two symbols are the same or different. We instead employ symbols that have algebraic significance.

We consider an $\text{SHF}_\lambda(N; k, q^s, \{w_1, \dots, w_s\})$ whose symbols are column vectors from \mathbb{F}_q^s . The entry in row r and column c of the SHF is denoted by $\mathbf{v}_{r,c}$. Suppose that C_1, \dots, C_s is a set of disjoint sets of columns. Row r is *covering* for $\{C_1, \dots, C_s\}$ if, whenever we choose s columns $\{\gamma_i \in C_i : 1 \leq i \leq s\}$, the $s \times s$ matrix $[\mathbf{v}_{r,\gamma_1} \cdots \mathbf{v}_{r,\gamma_s}]$ is nonsingular over \mathbb{F}_q . Then the $\text{SHF}_\lambda(N; k, q^s, \{w_1, \dots, w_s\})$ is *covering* if, for every way to choose $\{C_1, \dots, C_s\}$, there are at least λ covering rows.

We establish that covering separating hash families of type $1^t d^1$ give an effective construction for detecting arrays, which are useful in screening complex systems to find interactions among t or fewer factors without being masked by d or fewer other interactions. This connection easily accommodates outlier and missing responses in the screening. We explore asymptotic existence results and explicit constructions using finite geometries for covering separating hash families. We develop randomized and derandomized construction algorithms and discuss consequences for detecting arrays. This is joint work with Violet R. Syrotiuk (ASU).