Chernoff approximation of operator semigroups and applications

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We present a method to approximate operator semigroups with the help of the Chernoff theorem. We discuss different approaches to construct Chernoff approximations for semigroups, generated by Markov processes, and for Schrödinger groups. This method provides simultaneousely some numerical schemes for PDEs and pseudo-differential equations (in particular, the operator splitting method), Euler–Maruyama schemes for the corresponding SDEs and other Markov chain approximations to the corresponding Markov processes, can be understood as a numerical path integration method. In some cases, Chernoff approximations have the form of limits of n iterated integrals of elementary functions as $n \to \infty$ (in this case, they are called *Feynman formulae*) and can be used for direct computations and simulations of stochastic processes. The limits in Feynman formulae sometimes coincide with (or give rise to) path integrals with respect to probability measures (such path integrals are usually called *Feynman-Kac formulae*) or with respect to Feynman type pseudomeasures. Therefore, Feynman formulae can be used to approximate the corresponding path integrals and to establish relations between different path integrals.

In this talk, we discuss Chernoff approximations for semigroups generated by Feller processes in $\mathbb{R}^d$. We are also interested in constructing Chernoff approximations for semigroups, generated by Markov processes which are obtained by different operations from some original Markov processes. In this talk, we discuss Chernoff approximations for such operations as: a random time change via an additive functional of a process, a subordination (i.e., a random time change via an independent a.s. nondecreasing 1-dim. Lévy process), killing of a process upon leaving a given domain, reflecting of a process. These results allow, in particular, to obtain Chernoff approximations for subordinate diffusions on strar graphs and compact Riemannian manifolds. Moreover, the constructed Chernoff approximations for evolution semigroups can be used further to approximate solutions of some time-fractional evolution equations describing anomalous diffusion (solutions of such equations do not posess the semigroup property and are related to some non-Markov processes).

References

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