

The Slow-Coloring Game on a Graph

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The *slow-coloring game* is played by Lister and Painter on a graph G . Initially, all vertices of G are uncolored. In each round, Lister marks a non-empty set M of uncolored vertices, and Painter colors a subset of M that is independent in G . The game ends when all vertices are colored. The score of the game is the sum of the sizes of all sets marked by Lister. The goal of Painter is to minimize the score, while Lister tries to maximize it; the score under optimal play is the *cost* of the graph.

A greedy strategy for Painter keeps the cost of G to at most $\chi(G)n$ when G has n vertices, which is asymptotically sharp for Turán graphs. On various classes Painter can do better. For n -vertex trees the maximum cost is $\lfloor 3n/2 \rfloor$. There is a linear-time algorithm and inductive formula to compute the cost on trees, and we know all the extremal n -vertex trees. Also, Painter can keep the cost to at most $(1 + 3k/4)n$ when G is k -degenerate, $7n/3$ when G is outerplanar, $3.9857n$ when G is acyclically 5-colorable, and $3.4n$ when G is planar. These bounds are not believed to be sharp.

We will discuss strategies (algorithms) for Lister and Painter that establish various lower and upper bounds. The results appear in three papers with various subsets of Grzegorz Gutowski, Tomasz Krawczyk, Thomas Mahoney, Krzysztof Maziarczyk, Gregory J. Puleo, Michal Zajac, and Xuding Zhu.