Parallel Newton-Chebyshev Polynomial Preconditioners for the Conjugate Gradient method

Ángeles Martínez

Department of Mathematics and Earth Sciences, University of Trieste

amartinez@units.it

Luca Bergamaschi

Department of Civil Environmental and Architectural Engineering University of Padua

luca.bergamaschi@unipd.it

Discretization of PDEs modeling different processes and constrained/unconstrained optimization problems often require the repeated solution of large and sparse linear systems $A\mathbf{x} = \mathbf{b}$. The size of these system can be of order $10^6 \div 10^9$ and this calls for the use of iterative methods, equipped with ad-hoc preconditioners as accelerators running on a parallel computing environment. In most cases, the huge size of the matrices involved prevents their complete storage. In these instances only the application of the matrix to a vector is available as a routine (*matrix -free regime*). Differently from direct factorization methods, iterative methods do not need the explicit knowledge of the coefficient matrix. The issue is the construction of a preconditioner which also work in a matrix-free regime. Polynomial preconditioners, i.e. preconditioners that can be expressed as $P_k(A)$, are very attractive for several reasons i.e. their construction is only theoretical, namely only the coefficients of the polynomial are to be computed with negligible computational cost, the application of $P_k(A)$ requires a number, k, of matrix-vector products so that they can be implemented in a matrix-free regime, and the eigenvectors of the preconditioned matrix are the same as those of A.

We consider polynomial preconditioners to accelerate the Conjugate Gradient method in the solution of large symmetric positive definite linear systems in massively parallel environments. We put in connection a specialized Newton method to solve the matrix equation $X^{-1} = A$ [1] and the Chebyshev polynomials for preconditioning. We propose a simple strategy to avoid clustering of the extremal eigenvalues in order to speed-up convergence. Numerical results on very large linear systems (up to 8 billion unknowns) in a parallel environment show the efficiency of the proposed class of preconditioners.

References

[1] L. Bergamaschi and A. Martínez, *Parallel Newton-Chebyshev polynomial preconditioners for the Conjugate Gradient method*, Computational and Mathematical Methods, (2021). to appear.