Very weak solutions to PDEs in inhomogeneous and anisotropic spaces

Iwona Chlebicka
University of Warsaw
i.chlebicka@mimuw.edu.pl

I will discuss well-posedness and regularity to nonlinear PDEs of simple divergent form

$$-\mathrm{div} A(x, \nabla u) = f \quad \text{or} \quad \partial_t u - \mathrm{div} A(t, x, \nabla u) = f,$$

where the datum f is merely integrable or even is a measure, whereas the growth of A is governed by an inhomogeneous and fully anisotropic N-function $M(x, \nabla u)$. Inhomogeneity means space-dependence and anisotropy yields dependence of M on ∇u not necessarily via its length $|\nabla u|$.

The datum is too poorly regular for weak solutions to exist, thus a more delicate notion of very weak solutions is necessary. Besides existence and uniqueness they share some regularity properties of the weak ones. The generality we admit covers classical linear and polynomial growth operators possibly with measurable coefficients, generalizations of p-Laplacian involving log-Hölder continuous variable exponent, as well as problems posed in fully anisotropic Orlicz spaces (under no growth conditions), and double-phase spaces within the range of parameters sharp for density of smooth functions. The conditions for the density and their importance will be stressed.

The talk will be based on series of joint papers with: Youssuf Ahmida, Angela Alberico, Andrea Cianchi, Piotr Gwiazda, Ahmed Youssfi, and Anna Zatorska-Goldstein.