Classifying compact PL 4-manifolds according to generalized regular genus and G-degree

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(d+1)-colored graphs, that is $(d+1)$-regular graphs endowed with a proper edge-coloration, are the objects of a long-studied representation theory for closed PL d-manifolds, which has been recently extended to the whole class of compact PL d-manifolds.

In this context, combinatorially defined PL invariants play a relevant role; in this talk we will focus on two of them: the *generalized regular genus* and the *G-degree*. The former extends to higher dimension the classical notion of Heegaard genus for 3-manifolds; the latter has arisen in connection with *Colored Tensor Models* (CTM), a particular kind of tensor models, that have been intensively studied in the last years, mainly as an approach to quantum gravity in dimension greater than two. CTMs established a link between colored graphs and tensor models, since the Feynman graphs of a d-dimensional CTM are precisely $(d+1)$-colored graphs. Furthermore, the G-degree of a colored graph is a crucial quantity driving the 1/N expansion of the free energy of a CTM.

This talk will mainly concern recent results achieved in dimension 4: in particular, the classification of all compact PL 4-manifolds with generalized regular genus at most one or with G-degree at most 18. Furthermore, we will discuss interesting classes of 5-colored graphs (*semi-simple and weak semi-simple crystallizations*), representing compact PL 4-manifolds with empty or connected boundary and minimizing the invariants. In the simply-connected case they also belong to a wider class of 5-colored graphs which are proved to induce handle-decompositions of the represented 4-manifold lacking in 1- and/or 3-handles; therefore their study is strictly related to the problem, posed by Kirby, of the existence of special handle-decompositions for any simply-connected closed PL 4-manifold.