

Reverse superposition estimates, lifting over a compact covering and extensions of traces for fractional Sobolev mappings

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When $u \in W^{1,p}(\mathbb{R}^m)$, then $|u| \in W^{1,p}(\mathbb{R}^m)$ and

$$\int_{\mathbb{R}^m} |Du|^p = \int_{\mathbb{R}^m} |D|u||^p;$$

this provides an a priori control on u by $|u|$ in first-order Sobolev spaces. For fractional Sobolev spaces when $sp > 1$, we prove a reverse oscillation inequality that yields a control on u by $|u|$ in $W^{s,p}(\mathbb{R}^m)$. As another consequence of the reverse oscillation estimate, given a covering map $\pi : \tilde{\mathcal{N}} \rightarrow \mathcal{N}$, with $\tilde{\mathcal{N}}$ compact, we prove any $u \in W^{s,p}(\mathbb{R}^m, \mathcal{N})$ has a lifting, that is, can be written as $u = \pi \circ \tilde{u}$, with $\tilde{u} \in W^{s,p}(\mathbb{R}^m, \tilde{\mathcal{N}})$. This completes the picture for lifting of fractional Sobolev maps and implies the surjectivity of the trace operator on Sobolev spaces of mappings into a manifold when the fundamental group is finite.