Linear maps which are (triple) derivable or anti-derivable at a point

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A typical challenge in the setting of preservers asks whether a linear map T from a C*-algebra A into a Banach A-bimodule X behaving like a derivation (i.e. $D(ab) = D(a)b + aD(b)$) or like an anti-derivation ($D(ab) = D(b)a + bD(a)$) only on those pairs of elements (a, b) in a proper subset $\mathfrak{D} \subset A^2$ is in fact a derivation or an anti-derivation. A protagonist role is played by sets of the form $\mathfrak{D}_z := \{(a, b) \in A^2 : ab = z\}$, where z is a fixed point in A. A linear map $T : A \to X$ is said to be a *derivation* or an *anti-derivation at a point* $z \in A$ if it behaves like a derivation or like an anti-derivation on pairs $(a, b) \in \mathfrak{D}_z$. These maps are usually called *derivable or anti-derivable at* z. Let us simply observe that applying a similar method to define linear maps which are homomorphisms at zero, we find a natural link with the fruitful line of results on zero products preservers.

A recent study developed by B. Fadaee and H. Ghahramani in [3] characterizes continuous linear maps from a C*-algebra A into its bidual which are derivable at zero. A similar problem was considered by H. Ghahramani and Z. Pan for linear maps on a complex Banach algebra which is zero product determined [4]. These authors also find necessary conditions to guarantee that a continuous linear map $T : A \to A^{**}$ is anti-derivable at zero, where A is a C*-algebra, and also for linear maps on a zero product determined unital *-algebra to be anti-derivable at zero.

We have been involved in the study of those linear maps on C*-algebras which are derivations or triple derivations at zero or at the unit [2]. We shall revisit some of the main conclusions on these kind of maps from the perspective of preservers. We have further explored in [1] whether a full characterization of those (continuous) linear maps on a C*-algebra which are (*-)anti-derivable at zero can be given in pure algebraic terms. In this talk we shall present the latest advances in [1], which provide a complete solution to this problem.

References

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