

On maximal cliques in Paley graphs of square order

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In [1], Blokhuis studied maximum cliques in Paley graphs of square order $P(q^2)$. It was shown that a clique of size q in $P(q^2)$ is necessarily a quadratic line in the corresponding affine plane $A(2, q)$.

Let $r(q)$ denote the remainder after division of q by 4. In [2], for any odd prime power q , a maximal (but not maximum) clique in $P(q^2)$ of size $\frac{q+r(q)}{2}$ was constructed.

In [3], for any odd prime power q , a maximal clique in $P(q^2)$ of the same size $\frac{q+r(q)}{2}$ was constructed. This clique was shown to have a remarkable connection with eigenfunctions of $P(q^2)$ that have minimum cardinality of support $q + 1$.

In this talk, we discuss the constructions of maximal cliques from [2] and [3] and establish a correspondence between them.

Acknowledgments. Sergey Goryainov and Leonid Shalaginov are supported by RFBR according to the research project 20-51-53023.

References

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