Discrete Fuglede conjecture on cyclic groups

Gergely Kiss

Alfréd Rényi Institute of Mathematics

kigergo57@gmail.com

Gábor Somlai

Eötvös Loránd University

zsomlei@gmail.com

Romanos Diogenes Malikiosis

Aristotle University of Thessaloniki

rwmanos@gmail.com

Máté Vizer

Alfréd Rényi Institute of Mathematics

vizermate@gmail.com

Fuglede in 1974 conjectured that a bounded domain $S \subset \mathbb{R}^d$ tiles the d-dimensional Euclidean space if and only if the set of functions in $L^2(S)$ admits an orthogonal basis of exponential functions.

In my talk we focus on the discrete version of Fuglede's conjecture that can be formulated as follows. A subset S of a finite abelian group G tiles G if and only if the character table of G has a submatrix, whose rows are indexed by the elements of S, which is a complex Hadamard matrix. Fuglede's original conjecture were disproved first by Tao and the proof is based on a counterexample on elementary abelian p-groups.

On the other hand, it is still an open question whether the discrete Fuglede's conjecture is true on cyclic groups. In my talk I will summarize the known results concerning this question. In particular, I will present our recent result which shows that the conjecture holds on cyclic groups whose order is the product of at most 4 (not necessarily different) primes. I will introduce a geometric technique that we called 'cube-rule' and which is an essential tool of the proof.