

Measure of noncompactness of Sobolev embeddings

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A bounded set can be covered by a single ball of some radius. Sometimes several balls of a smaller radius can also cover the set. A compact set can be covered by finitely many balls of arbitrary small radius. The smallest radius that allows to cover the set with finitely many balls therefore describes sets laying in between boundedness and compactness. Such quantity is called a measure of non-compactness.

Based on the property of images of the unit balls, linear mappings between Banach spaces are also classified as bounded or compact and to those staying in between, we can assign the measure of non-compactness as well.

An important instance of an operator is a Sobolev embedding. Compactness of a Sobolev embedding can constitute a crucial step in many applications in partial differential equations, probability theory, calculus of variations, mathematical physics and other disciplines. In the non-compact case, more subtle techniques have to be developed and the measure of non-compactness plays an indispensable role here.

We give a survey of some recent new results on measure of non-compactness of Sobolev embeddings and related mappings.