Core reduction: Necessary and sufficient information in linear approximation problems

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We focus on linear approximation problems $Ax \approx b$, where A is a given matrix, x an unknown vector, and the given right-hand side vector b is not in the range of A, i.e., $b \notin R(A)$. By solving of such problem we usually mean replacing it by some minimization. Typically the least squares (LS) techniques can be used. We focus on the so-called total least squares (TLS) minimization

$$\min \|[g, E]\|_F \quad \text{s.t.} \quad (b+g) \in R(A+E).$$

TLS has been studied since the early eighties. The trouble there is, contrary to the standard LS, that the minimization may not have a solution for the given (A, b).

The theory of core problem introduced in 2006 by Paige and Strakoš brings a concept of necessary and sufficient information for solving the TLS minimization. This concept allows us to distinguish cases having and not having the TLS solution. Moreover, core problem theory clearly explains why it happens. In recent years the core problems thoery has been applied on several other linear problems $A(X) = B$ where the linear mapping A as well as the right-hand side B can have some particular structure.