Universality classes of triangulations in dimensions greater than 2

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Combinatorial maps are cellular topological models for surfaces, which include triangulations, quadrangulations, etc. of all genera. It is well-known that at fixed topology, all models lie in the same universality class. It means in particular that they have the same critical exponents in the enumeration formula and the same limit object at large scales. For instance, the universality class for planar maps is also known as the universality class of 2D pure quantum gravity and its scaling limit is known as the Brownian sphere. It is important to generalize this framework to greater dimensions, with motivations from combinatorics, topology, mathematical physics and quantum gravity. However, it has been a challenge due to the intricate nature of combinatorics and topology in dimensions 3 and greater. Here I will present results based on the model of colored triangulations for PL-manifolds. They admit a representation as edge-colored graphs which is amenable to combinatorial analysis. I will focus on a combinatorial generalization of Euler's relation for combinatorial maps, which is to bound the number of $(d-2)$-simplices linearly in the number of d-simplices, and identifying the triangulations which maximize the bound for different colored building blocks. In 3d, I prove that those triangulations, for any set of colored building blocks homeomorphic to the 3-ball, are in bijection with trees. In even dimensions, we have proved that several universality classes can be achieved. Both those results are in sharp contrast with the case of surfaces.