On the rank of pseudo walk matrices

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Abstract

In the literature, the walk matrix $\mathbf{W_b}$ associated with a graph G having vertex set $\mathcal{V}(G)$ is the matrix with columns $\mathbf{b}, \mathbf{Ab}, \mathbf{A}^2\mathbf{b}, \ldots, \mathbf{A}^{r-1}\mathbf{b}$ that enumerates the number of all possible walks on G of length $0, 1, 2, \ldots, r-1$ starting from each vertex of G and ending at any of the vertices indicated by $\mathbf{b}$. We generalize walk matrices further to obtain pseudo walk matrices $\mathbf{W_v}$ having any walk vector $\mathbf{v}$. For any subset S of $\mathcal{V}(G) \times \mathcal{V}(G)$, the total number of walks $N_0(S), N_1(S), N_2(S), \ldots$ of length $0, 1, 2, \ldots$ in G that start from vertex i and end at vertex j for all $(i, j) \in S$ is considered. Various results on such pseudo walk matrices are presented, particularly related to their rank. The matrix rank of pseudo walk matrices allows the consideration of controllable and recalcitrant pairs.