On a weighted inequality for fractional integrals

Giorgi Imerlishvili
Georgian Technical University
imerlishvili18@gmail.com
Alexander Meskhi
Kutaisi International University, Georgia
alexander.meskhi@kiu.edu.ge

The weighted inequality

$$\|I_\alpha f\|_{L^q_V} \leq C\|f\|_{L^p} \tag{1}$$

for the Riesz potential I_α, $0 < \alpha < n$, plays an important role in the theory of PDEs. It is worth mentioning its applications to the theory of Sobolev embeddings (see, e.g., [Maz]), its connection with eigenvalue estimates for the Schrödinger operator $H = -\Delta - V$ with a potential V (see, e.g., [FJW], PP. 91-94), etc. In 1972 D. Adams proved that the above mentioned weighted inequality holds for $1 < p < q < \infty$ if and only if there is a positive constant C such that for all balls B in $\mathbb{R}^n$,

$$V(B) \leq C|B|^{(1/p-\alpha/n)q}.$$

In the diagonal case, i.e., when $p = q$, necessity of this condition remains valid, however, it is not sufficient for (1) (see, e.g., [Le]). We proved that the condition

$$V(B) \leq C|B|^{1-p\alpha/n}$$

is simultaneously necessary and sufficient for the boundedness of I_α from the Lorentz space $L^{p,1}$ to the weighted Lebesgue space L^p_V. Some other related results are also derived.

Acknowledgment. The work was supported by the Shota Rustaveli National Foundation grant of Georgia (Project No. DI-18-118).

References

[FJW] M. Frazier, B. Jawerth and G. Weiss, Littlewood-Paley theory and the study of function spaces, *Regional Conference Series in Mathematics*, Vol. 79, *Amer. Math. Soc. Providence. Rl,* 1991.

[Le] P.-G. Lemarié, Multipliers and Morrey spaces, *Potential Analysis*, **38**(2013), 741-752.

[Maz] V. Maz'ya, Sobolev spaces, *Springer, Berlin*, 1984.