Meet preservers between lattices of real-valued continuous functions

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It is well-know that the set $C(X)$ of real-valued continuous functions defined on a compact Hausdorff space X becomes a lattice when equipped with the usual point-wise ordering; in particular, the join and meet of $f, g \in C(X)$ are given by

$$(f \vee g)(x) = \max\{f(x), g(x)\} \quad \text{and} \quad (f \wedge g)(x) = \min\{f(x), g(x)\},$$

respectively. We will demonstrate that any surjective $T\colon C(X) \to C(Y)$ satisfying

$$\mathrm{Ran}_\pi(f \wedge g) = \mathrm{Ran}_\pi(T(f) \wedge T(g))$$

for all $f, g \in C(X)$, where $\mathrm{Ran}_\pi(\cdot)$ denotes the set of range values of maximum absolute value, induces a homeomorphism $\psi\colon Y \to X$ such that

$$T(f) = f \circ \psi$$

holds for all $f \in C(X)$ with $0 \leq f$.