On a problem of M. Talagrand

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We will discuss some special cases of a conjecture of M. Talagrand relating two notions of "threshold" for an increasing family $\mathcal{F}$ of subsets of a finite set X. The full conjecture implies equivalence of the "Fractional Expectation-Threshold Conjecture," due to Talagrand and recently proved by Frankston, Kahn, Narayanan, and myself, and the (stronger) "Expectation-Threshold Conjecture" of Kahn and Kalai.

The conjecture under discussion here says there is a fixed J such that if, for a given increasing family $\mathcal{F}$, $p \in [0,1]$ admits $\lambda : 2^X \to \mathbb{R}^+$ with

$$\sum_{S \subseteq F} \lambda_S \geq 1 \quad \forall F \in \mathcal{F}$$

and

$$\sum_S \lambda_S p^{|S|} \leq 1/2,$$

then p/J admits such a λ taking values in $\{0,1\}$.

Talagrand showed this when λ is supported on singletons and suggested a couple of more challenging test cases. In the talk, I will give more detailed descriptions of this problem, and some proof ideas if time allows.