Observability for Non-Autonomous Systems

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We study non-autonomous abstract Cauchy problems

$$\dot{x}(t) = A(t)x(t)\,,\ y(t) = C(t)x(t)\,,\ t > 0\,, \quad x(0) = x_0 \in X\,,$$

where $A(t) : D(A) \to X$ is a strongly measurable family of operators on a Banach space X and $C(t) \in \mathcal{L}(X,Y)$ is a family of bounded observation operators from X to a Banach space Y.

For measurable subsets $E \subseteq (0,T)$, $T > 0$, we provide sufficient conditions such that the Cauchy problem satisfies a *final state observability estimate*

$$\|x(T)\|_X \lesssim \left(\int_E \|y(t)\|_Y^r \,\mathrm{d}t \right)^{1/r}, \quad r \in [1,\infty)\,,$$

where an analogous estimate holds for the case $r = \infty$.

An application of the above result to families of strongly elliptic differential operators $A(t)$ and observation operators

$$C(t)u := \mathbf{1}_{\Omega(t)}u\,, \quad \Omega(t) \subseteq \mathbb{R}^d\,, u \in \mathrm{L}^p(\mathbb{R}^d)\,,$$

is presented. In this setting, we give sufficient and necessary geometric conditions on the family of sets $(\Omega(t))$ such that the corresponding Cauchy problem satisfies a final state observability estimate.