Research (Robert J. Berman)

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In general terms Berman's research has focused on the application of analytic techniques in complex algebraic geometry and complex differential geometry. In a nutshell this amounts to "doing calculus" on complex algebraic varieties, i.e. complex manifolds which may be embedded as the points cut out by homogeneous polynomials on complex projective space. Since the projective space comes with a natural line bundle (the hyperplane line bundle) such a projective embedding decorates the complex manifold X with a specific line bundle L which is positive. This leads to a close link to differential geometry, originating in the isomorphism between the space of positively curved metrics on L and the space of Kähler metrics on X, representing the cohomology class defined by the first Chern class of the line bundle L. In this field there is an attractive blend of concrete and abstract problems, ranging from concrete problems about interpolation and sampling of large degree polynomials to rather abstract problems linking highly non-linear PDEs to the differential and algebraic geometry of complex algebraic varieties. A major part of Berman's research has been devoted to problems motivated by the interactions between the concrete and abstract aspects. In particular, his research has been motivated by two central conjectures in pluripotential theory/interpolation theory and complex geometry, respectively: (1) Siciak's conjecture from the 80's about the equidistribution of higher dimensional Fekete points (which arise as optimal nodes for interpolating polynomials of a large degree) and (2) The Yau-Tian-Donaldson conjecture from the 90's about the existence of canonical Kähler metrics on a complex manifold X (i.e. Kähler-Einstein metrics, or, more generally, Kähler metrics with constant scalar curvature in the first Chern class of L). The first conjecture was settled in a joint work with Boucksom and Witt-Nyström and the second one is still open, in general, while the Kähler-Einstein some years ago, using a PDE method of continuity, combined with Gromov-Hausdorff convergence theory. A different variational proof was then given by Berman-Boucksom-Jonsson, using, in particular, the convexity of the K-energy functional established by Berman-Berndtsson. In the last years Berman has introduced a unified statistical mechanical approach connecting Siciak's conjecture with the Yau-Tian-Donaldson conjecture, by bringing probability into the picture. In this approach the limiting distribution of Fekete points in Siciak's conjecture appears as the macroscopic equilibrium state at zero temperature, while the canonical metric in the

Yau-Tian-Donaldson conjecture should appear at non-zero temperature. This means that the canonical metrics in question could be constructed by a direct sampling procedure using Monte-Carlo methods. The statistical mechanical approach also ties in naturally with the variational approach to complex Monge-Ampère equations and Kähler-Einstein metrics that Berman has developed in colaborations with Boucksom, Guedj, Eyssideux and Zeriahi.