

I have been working on a variety of different topics in Combinatorics and Theoretical Computer Science. For this document I have selected those two areas to which my most recent research pertains.

Propositional Proof Complexity

Proof complexity is an interdisciplinary area bordering complexity theory on one side and mathematical logic on the other. It studies *efficient*, often called *feasible*, provability of various statements of interest in formal proof systems. The emphasis on efficiency, as opposed to mere *existence*, is what makes proof complexity different from its parent discipline, classical proof theory.

Propositional proof complexity further forbids quantifiers thereby confining us to the statements of purely discrete and finite nature. Despite this restriction, it turns out that a great deal of very interesting problems both in theory (notably combinatorial optimization) and practice (hardware/software verification being paradigmatic example) can be perfectly formalized in this restricted language. Moreover, most methods employed for solving concrete problems in those areas have faithful mathematical counterparts in the theoretical proof complexity. That makes the latter well-connected to many surprisingly distant things.

Core questions of proof complexity are the same as in other complexity theories. Upper bounds: what true statements have efficient (say, short) proofs? How to show that some statements do not possess any such proof (lower bounds)?

For highly informal essay on mathematical aspects of proof complexity see [1] and for connections to the practical SAT solving see [2].

Continuous Combinatorics

This is a loose generic term for several related approaches to studying those properties of huge combinatorial structures that are resilient to small changes: examples include ordinary graphs, coloured tournaments or partial orders. What is a “good” definition of distance between such structures and whether it leads to interesting notions of convergence, compactification and topology? In what terms can we characterize the resulting limit objects up to an isomorphism, defining first what the latter means? It turns out that the key to answering these questions lies in the statistical properties of the structure or, slightly more precisely, what is the distribution on fixed size structures we can see in a small window sampled from the original structure at random. Then the description of limit objects can be done in many equivalent ways:

geometric, algebraic, in the language of ergodic theory, in terms of exchangeable arrays etc. Besides, studying sampling statistics “for their own sake” is a recurrent theme in classical extremal combinatorics, and this continuous point of view has led to many concrete results in the latter area.

For a comprehensive text on graph limits (predominantly geometric story tailored to the case of ordinary graphs) see [3]. For flag algebras (largely algebraic approach to arbitrary structures based on the first-order logic) and a somewhat outdated list of their concrete applications see [4, 5]. The preprint [6] is a very recent attempt at their unification.

References

- [1] A. Razborov. Proof complexity and beyond. *SIGACT News*, 47(2):66–86, 2016.
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- [3] L. Lovász. *Large Networks and Graph Limits*. American Mathematical Society, 2012.
- [4] A. Razborov. Flag algebras. *Journal of Symbolic Logic*, 72(4):1239–1282, 2007.
- [5] A. Razborov. Flag algebras: an interim report. In *Mathematics of Paul Erdős*, pages 207–232. Springer-Verlag, 2nd edition, 2013.
- [6] L. Coregliano and A. Razborov. Semantic limits of dense combinatorial objects. Available at <http://people.cs.uchicago.edu/~razborov/files/theons.pdf>, 2019.